Software System Design and Implementation

Existentially Quantified Types

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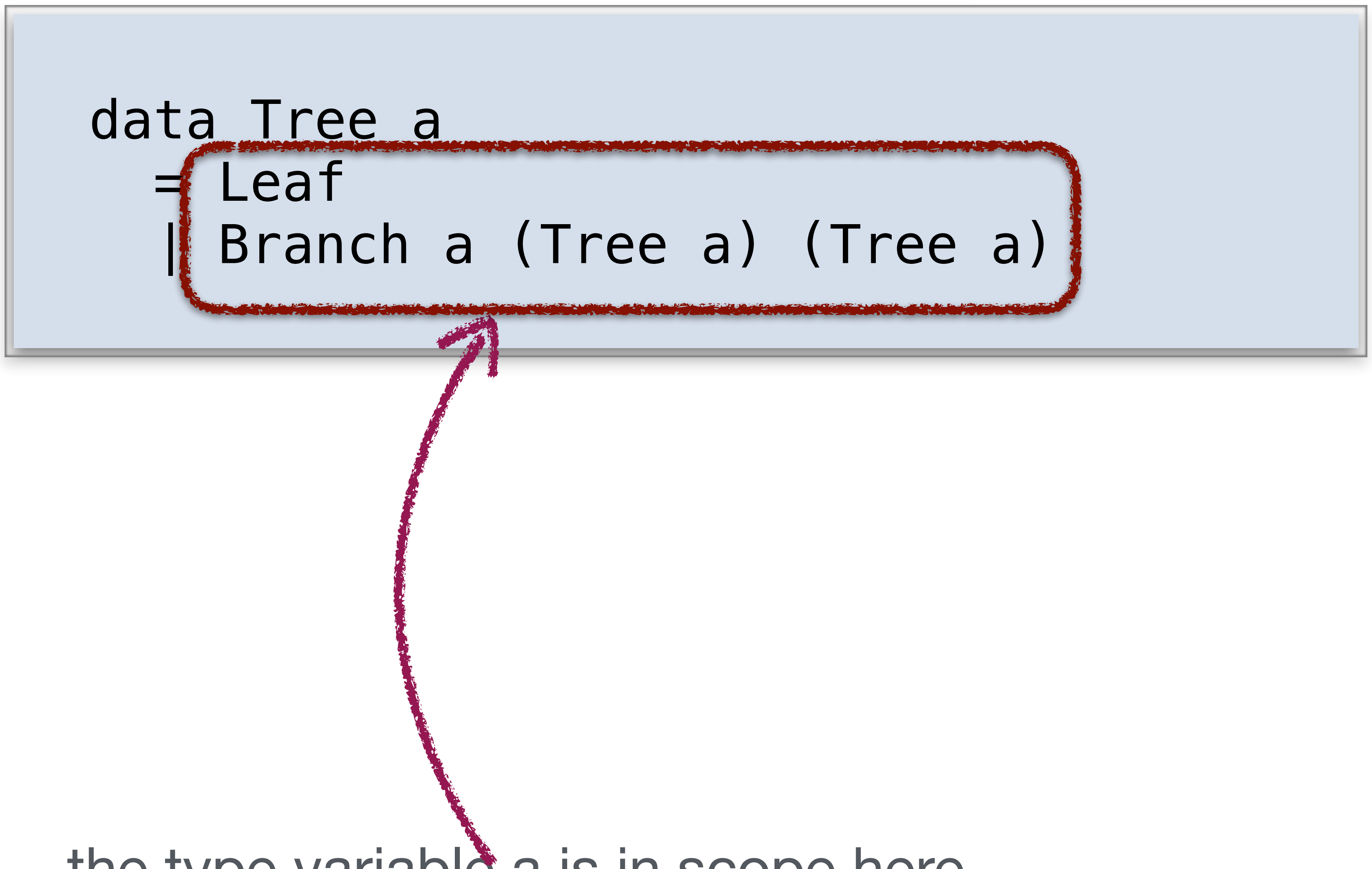
Scope of type variables



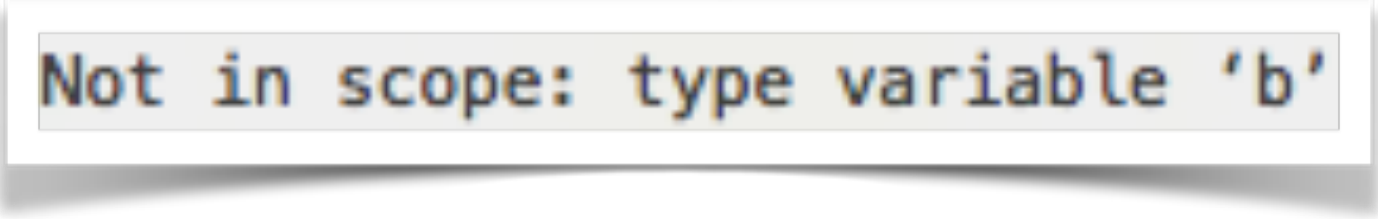
```
data Tree a
  = Leaf
  | Branch a (Tree a) (Tree a)
```

the type variable a is in scope here

Scope of type variables

we can only use type variables which are in scope

```
data Tree a
  = Leaf
  | Branch b (Tree b) (Tree b)
```



```
Not in scope: type variable ‘b’
```

Scope of type variables

but we don’t ***have*** to use them (phantom types):

```
data Length a  = Length Double

data Kilometer
data Miles

addLength :: Length a -> Length a -> Length a
addLength (Length n) (Length m)
  = Length (n + m)
```

Scope of type variables

With GADT notation:

```
data Tree a where
  Leaf   :: Tree a
  Branch :: a -> Tree a -> Tree a -> Tree a
```

which is equivalent to:

```
data Tree a where
  Leaf   :: Tree a
  Branch :: b -> Tree b -> Tree b -> Tree b
```

Scope of type variables

Type variables are implicitly ∀-quantified:

```
data Tree a where
  Leaf   :: forall a. Tree a
  Branch :: forall a. a -> Tree a -> Tree a -> Tree a
```

```
data Tree a where
  Leaf   :: forall a. Tree a
  Branch :: forall b. b -> Tree b -> Tree b -> Tree b
```

Scope of type variables

- Type variables don’t have to appear in the result

```
data M where
   MC :: a -> M
```

```
data M where
   MC :: forall a. a -> M
```

- or in non-GADT notation (needs language extension enabled)

```
data M = forall a. MC a
```

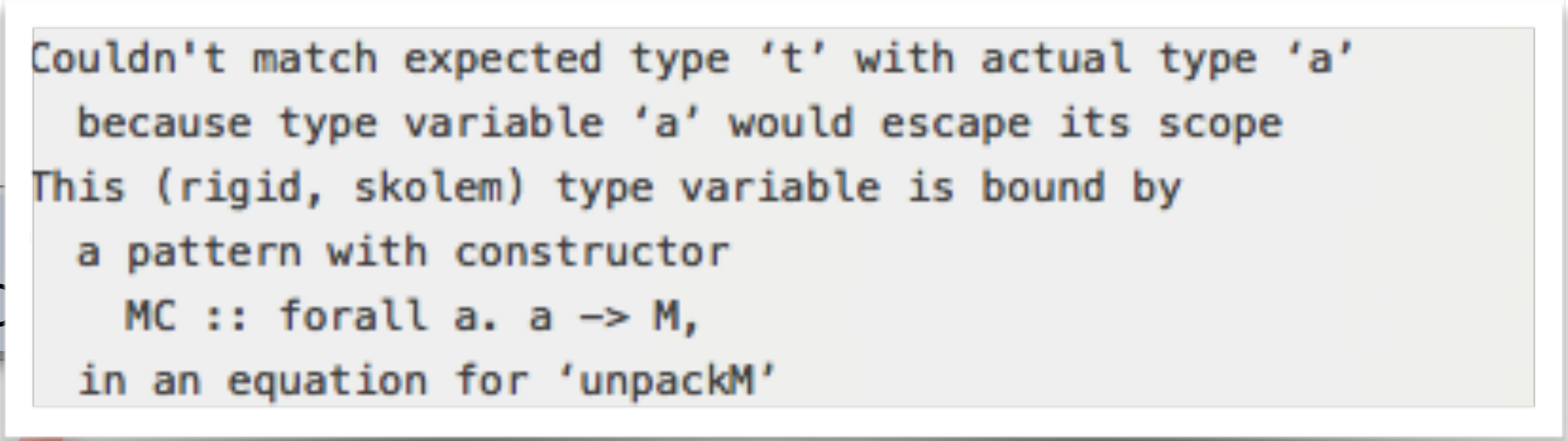
Scope of type variables

```
data M where
   MC :: a -> M
```

- We can define a list of values of type M:

```
xs :: [M]
xs = [MC 5, MC True, MC “Why??”]
```

```
unpackM ::
unpackM (MC
```



```
Couldn't match expected type ‘t’ with actual type ‘a’
  because type variable ‘a’ would escape its scope
This (rigid, skolem) type variable is bound by
  a pattern with constructor
    MC :: forall a. a -> M,
  in an equation for ‘unpackM’
```

There is nothing we can do with values of type M!

Existential Types

- So, what is the actual type of unpackM?

```
unpackM :: M -> a
```

- Recall that type variables in Haskell are implicitly ∀-quantified, so the above type is the same as

```
unpackM :: forall a. M -> a
```

- But the real type of unpackM is (which can't be expressed in Haskell):

```
unpackM ::  ∃a. M -> a
```

- This is why these types are called ‘existential types’

```
{-# LANGUAGE ExistentialQuantification #-}

data M = forall a. MC a
```

Existential Types

```
data N where
   NC :: Show a => a -> N
```

```
data P where
   PC :: (a -> String) -> a -> P
```

```
showNs :: [N] -> [String]
showNs ns = map show’ ns
  where
    show’ (NC x) = show x
```

```
showPs :: [P] -> [String]
showPs ps = map (\(PC f p) -> f p) ps
```

Example: Shapes

- Haskell:

```
data Shape
  = Circle …
  | Rectangle …
  | Square …

perimeter :: Shape -> Double
perimeter (Circle …) =
perimeter (Rectangle …) =
…

area :: Shape -> Double
…
```

- easy to add new functions on the Shape type, less so to add more variants

Example: Shapes

- In OO-languages
  - class Shape
  - Circle, Rectangle, Square extend the class
  - easy to add new variants, less so to add more functions
- Use classes and overloading to model this in Haskell?

```
class Shape a where
   perimeter :: a -> Double
   area      :: a -> Double

data Circle    = Circle …

instance Shape Circle where
  perimeter (Circle …) = …
  area      (Circle …) = …
```

Rank-n polymorphism

- Write a function which, given
  - a polymorphic list constructor function a -> [a]
  - and two values of possibly different types
  - applies this function to both values and returns the lists
- Is this function type correct?

```
foo f a b = (f a, f b)
```

- **Problem:** we can write polymorphic functions in vanilla Haskell, but we can express the fact that we want a polymorphic function as argument

Rank-n polymorphism

- **Problem:** we can write polymorphic functions in vanilla Haskell, but we can’t express the fact that we want a polymorphic function as argument

- Again, this is a scoping issue:

```
∀a. ∀b.(a -> [a]) -> a -> b -> ([a], [b])
```

versus

```
∀a. ∀b.(∀a. a -> [a]) -> a -> b -> ([a], [b])
```

Rank-n polymorphism

- **Rank-n polymorphism** makes this possible

```
∀a. ∀b.(∀a. a -> [a]) -> a -> b -> ([a], [b])
```

rank-2 polymorphic function

- **Rank-n polymorphism** can be used to control what information a function has access to

Remember the ST monad?

```
newSTRef   :: a -> ST s (STRef s a)
readSTRef  :: STRef s a -> ST s a
writeSTRef :: STRef s a -> a -> ST s ()

runST      :: (forall s. ST s a) -> a
```

Existential Types and Rank-n types

- Note the difference:

```
data M where
    MC :: a -> M
```

```
data M where
     MC :: forall a. a -> M
```

```
data M = forall a. MC a
```

vs

```
data M where
    MC :: (forall a.a) -> M
```

```
data M = MC (forall a. a)
```

Error Handling

- Two types of errors:
  - Fatal errors: indicates serious problems that an application should not try to catch, as it requires external fix: program bug, stack overflow…
  - Non-fatal errors: conditions that an application should catch and handle.
- Further distinction
  - Synchronous errors:
    - raised as a direct consequence by the program itself
  - Asynchronous errors:
    - timeouts, user interrupt, resource exhaustion

Asynchronous error handling

- Asynchronous errors can happen at any time
- Can’t (in general) be prevented from occurring by checks in the program
- Sometimes necessary to mask such exceptions to ensure proper clean-up

Synchronous error handling

- If a function can trigger a non-fatal error, it should in general be reflected in the type:

```
read         :: Read a => String -> a
readMaybe    :: Read a => String -> Maybe a
```

- If the function has to be partial for some reason, raise an appropriate error, don't just leave the patterns incomplete

- Compiler can detect incomplete patterns

```
-fwarn-incomplete-patterns
```

Synchronous error handling

- How errors are handled depends on programming language:
  - programming language support?
  - possible to throw exceptions?
  - exceptions declared in the type of a function/method?
  - handling statically enforced?